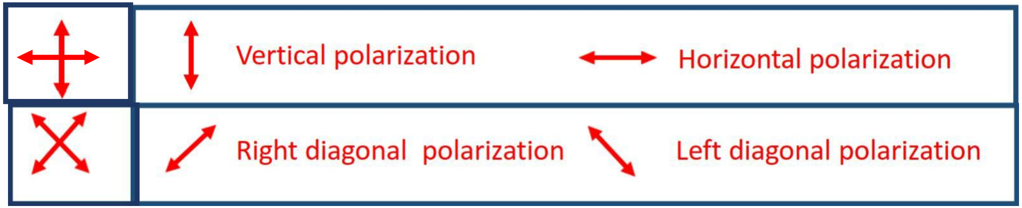
* Non-examinable discussion

Quantum measurement example: light polarization

- Light can be thought of as composed of elementary particles, which we call “photons”.
- A light wave, or an individual photon, can have a definite polarization, represented by an axis in space along which the wave oscillates.
- Quantum theory relates the possible polarization states by rules that depend on the angle between the axes.
- A measurement “asks” a photon to “choose” one of two orthogonal polarization states.



$$\psi = a\,\psi_{\updownarrow} + b\,\psi_{\leftrightarrow} \;;\; \psi_{\nearrow} = \frac{1}{\sqrt{2}}\left(\psi_{\updownarrow} + \psi_{\leftrightarrow}\right) \;;\; \psi_{\searrow} = \frac{1}{\sqrt{2}}\left(\psi_{\updownarrow} - \psi_{\leftrightarrow}\right)$$

The polarization states can be represented by 2D complex vectors:

$$\psi_{\updownarrow} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\psi_{\leftrightarrow} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\psi_{\swarrow\!\!\!\nearrow} = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$

$$\psi_{\nwarrow\!\!\!\searrow} = \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}.$$

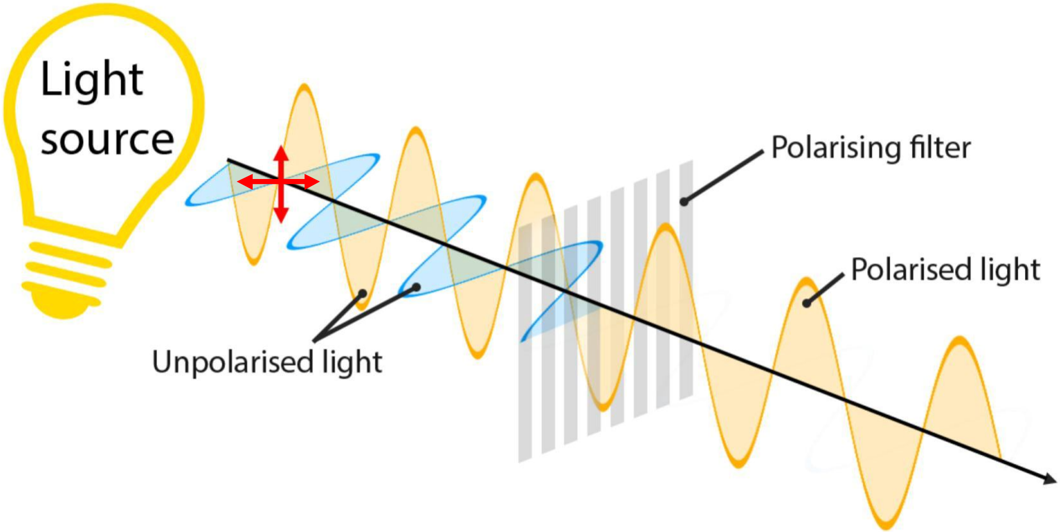
eigenvectors $\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Measuring "$\updownarrow$ or $\leftrightarrow$" corresponds to the operator $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

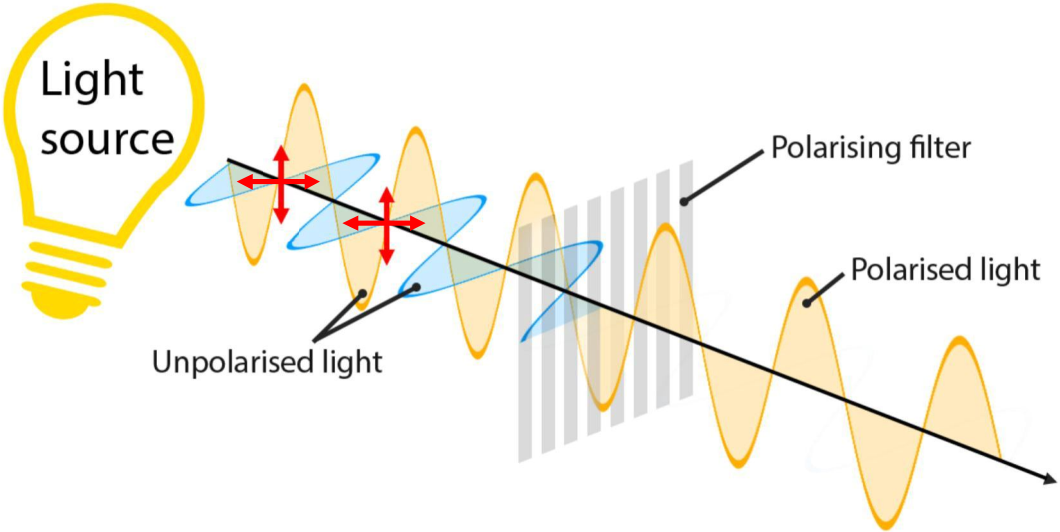
Measuring "$\swarrow\!\!\!\nearrow$ or $\nwarrow\!\!\!\searrow$" corresponds to the operator $\begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}$

eigenvectors $\begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}, \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}$

Vertical polarising filter blocks horizontally polarized photons and lets through vertically polarized photons



Vertical polarising filter blocks horizontally polarized photons and lets through vertically polarized photons



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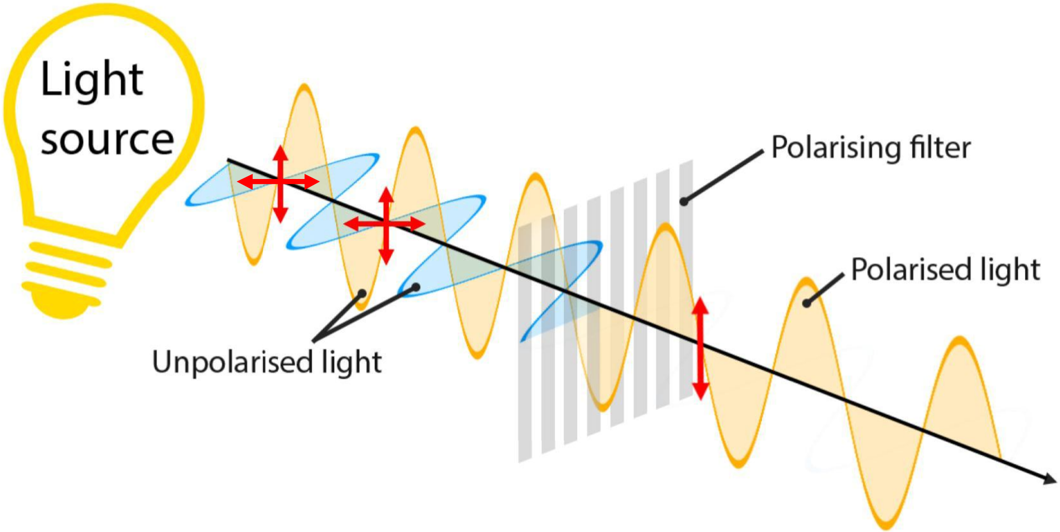
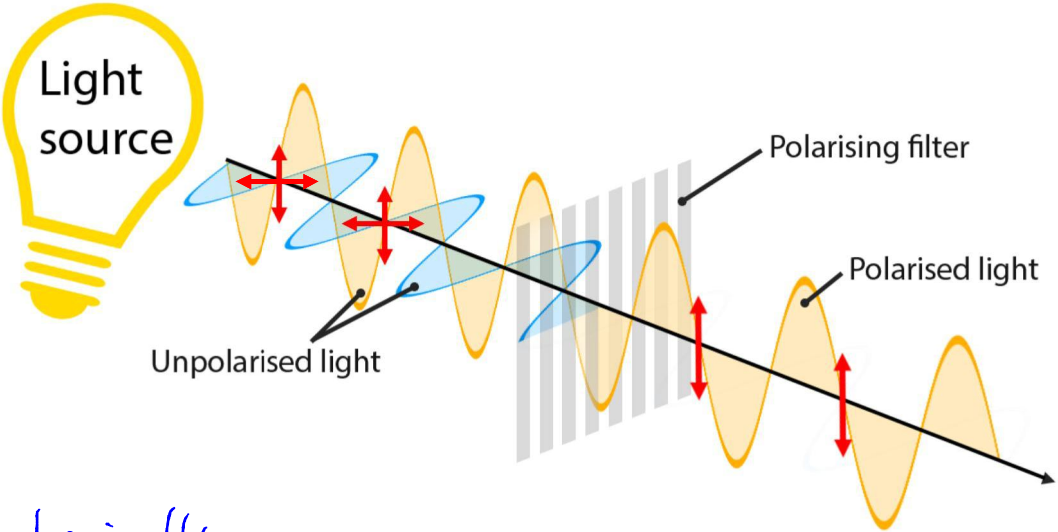


Vertical polarising filter blocks horizontally polarized photons and lets through vertically polarized photons



Quantum mechanically

$$\psi = a\,\psi_{\updownarrow} + b\,\psi_{\leftrightarrow} \xrightarrow[\text{postulate}]{\text{projection}} \psi_{\updownarrow} \text{ with probability } |a|^2.$$

$(|a|^2 + |b|^2 = 1)$

Quantum measurement rules

- Quantum theory tells us there is no way to directly ‘read’ a photon’s polarization state.
- We can **only** ‘ask’ binary questions, making the photon “choose” between two orthogonal states, e.g. “Are you vertically or horizontally polarized?”

- E.g. ask ⤢ “Are you ↕ or ↔ ?”

  “I am ↕ , sir.”

  “Are you sure? Were you ever anything else?”
  “Oh no, sir. I have always been ↕ .”

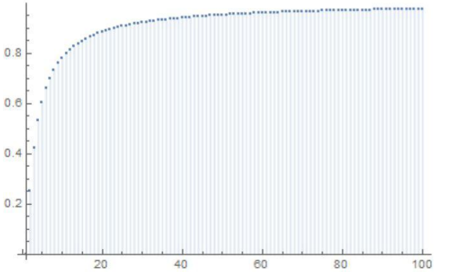
State	Measurement	Outcome Probabilities ↕	↔
↕	+	1	0
↔	+	0	1
⤢	+	0.5	0.5
⤡	+	0.5	0.5

Similarly for the alternative measurement ×

Quantum measurement as manipulation

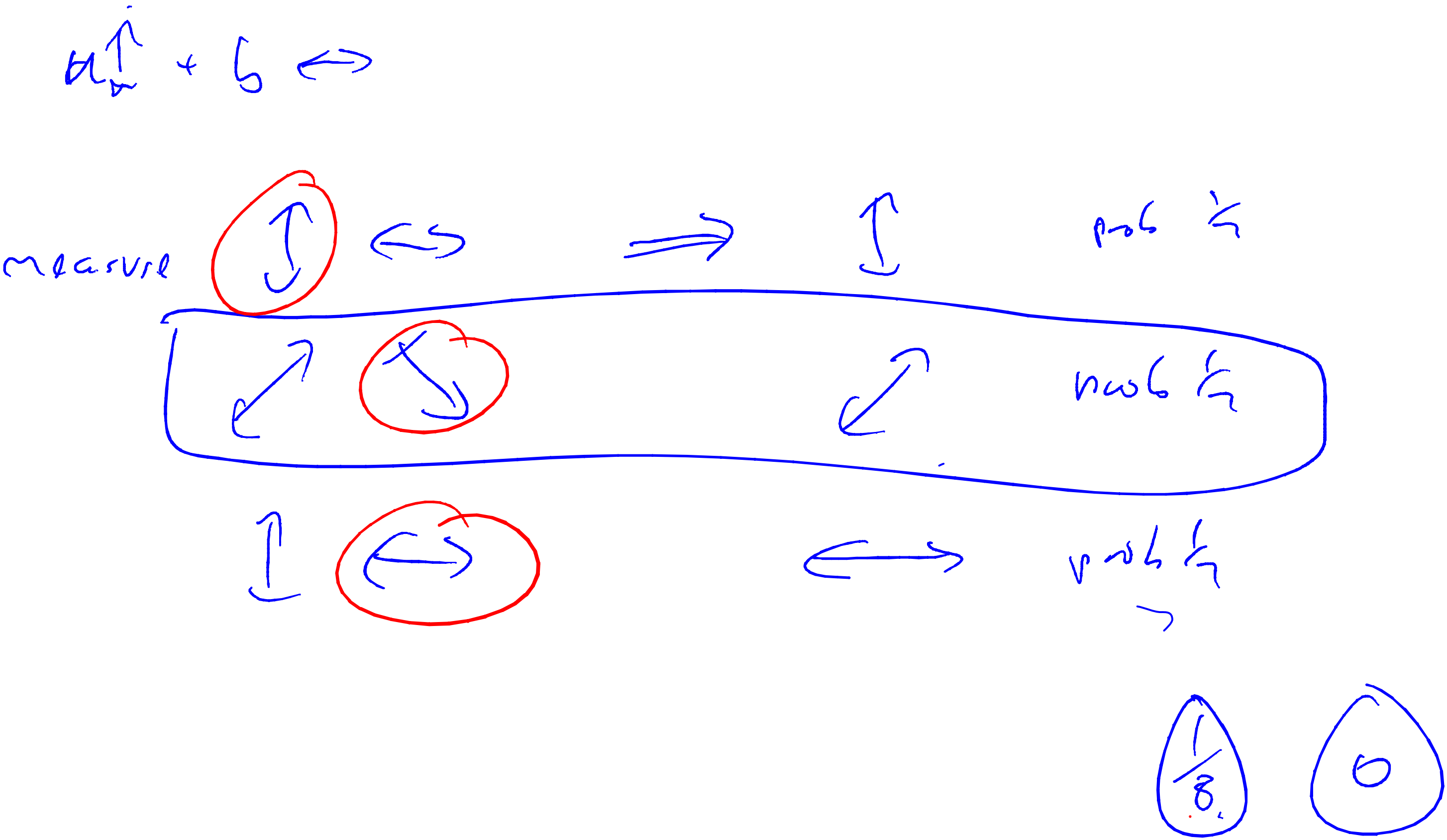
- We've seen that quantum measurements don't reveal a pre-existing state: they change the state.
- By choosing the right sequence of measurements, we can freeze a state that would otherwise be changing – the "quantum Zeno effect" – or *we can make a state behave in any way we wish.*
- E.g. we can change  to the opposite state  by carrying out a sequence of N measurements, with almost certain success for large N.

Success probability



Number of measurements (N)

(a stronger version of the Quantum Zeno Paradox, see Example sheet II.10)



* End of non-examinable discussion